AN APPLICATION OF THE GUIDE TO MEASUREMENT UNCERTAINTY

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Abstract - Do you understand how to apply the Guide to Measurement Uncertainty (GUM)?

The analysis supporting the Primary Standards Lab's accredited uncertainty claims for its 10MHz frequency reference is used to examine some of the differences in the various GUMs, the tradeoffs between using a simplified vs. a more rigorous approach, and how some of the difficult issues such as drift were handled.

INTRODUCTION

This analysis describes the uncertainty of the frequency of a Sulzer 2.5C ovenized oscillator and frequency doubler. It was chosen for presentation as a paper, not because it represents the state of the art for time and frequency measurements, rather because it contains some interesting aspects in the application of the GUM.

The oscillator is characterized by comparing its frequency to that of the 10 MHz oscillator internal to an HP 53503A global positioning (GPS) receiver. A PM6680B counter is used for the comparison by connecting its external reference input to the GPS receiver output and the counter input to the Sulzer oscillator. A system block diagram is shown in Figure 1.

The 10MHz House Standard is used to support the frequency calibration for Multifunction and Multiproduct calibrators, an AC Measurement Standard, and as the reference for a frequency counter in a Josephson Junction (JJ) Array system. An uncertainty analysis is necessary to support accredited claims of 1 part in 10^9 (10mHz) for the JJ.

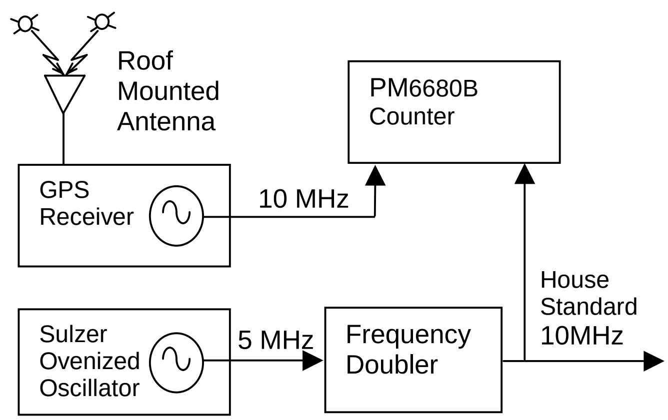

Figure 1: Block Diagram of GPS Referenced 10 MHz House Frequency Standard

If the analysis indicates much better performance for the system, tighter claims could be submitted for the calibration of frequency counters.

The output of the disciplined oscillator of the GPS receiver could be used directly for the House Standard. However, if the satellites were disabled, reception problems were encountered, or the receiver failed, the house clock would be unavailable. With the existing system, the GPS receiver is used to calibrate the Sulzer oscillator but, because of the excellent stability of the oscillator, it can be used as a check standard for the GPS system as well.

The uncertainty analysis was conducted according to five Guides to Uncertainty of Measurement (GUMs) [1-5]. Most of the requirements are very similar and are summarized in the checklist on the following page.

THE GUIDE TO UNCERTAINTY OF MEASUREMENT CHECKLIST

Singlas [3] and EA-4/02 [5] provide the better overviews of the uncertainty analysis process, a detailed flow chart and a step-by-step procedure respectively. The following is a composite checklist from those GUMs.

1. **Create the mathematical model, an equation describing the measured quantity as a function of each of the factors that can influence it.**

2. **Determine the estimated value for each of the input quantities, those factors in the equation above which can affect the measured, or output quantity.**

3. **Identify and apply all significant corrections.**

4. **List all the sources of error in the measurement process.**

5. **Write an uncertainty equation describing the errors in the measurement process.**

6. **Separate the uncertainty terms into Type A, those to be evaluated by direct observation, and Type B, those evaluated by other means.**

7. **For the Type A uncertainties:**

- **Perform the repeated measurements or sets of measurements.**
- **Calculate the standard deviation of the mean or pooled standard deviation from sets of readings.**
- **Calculate the effective degrees of freedom.**

8. **For the Type B uncertainties:**

- **Assign or estimate the uncertainty**
- **Assign the probability distribution**
- **Obtain or estimate the degrees of freedom**

9. **Calculate or assign the sensitivity coefficient associated with each of the input estimates.**

10. **Calculate or estimate the correlation coefficients or covariance between each of the input quantities with significant contributions.**

11. **Calculate the combined standard uncertainty for the measurement (output quantity).**

12. **Calculate the overall effective degrees of freedom.**

13. **Obtain the coverage factor, k, using a 95% confidence level and the effective degrees of freedom.**

14. **Calculate the expanded uncertainty**

15. **Report the expanded uncertainty and the coverage factor.**

1. MATHEMATICAL MODEL

The GUMs, and the bodies that accredit according to them, are becoming more insistent that the analysis begin with a mathematical model describing the output quantity Y as a function of N measurable quantities $X_1, X_2, \ldots, X_N$.

Eq. 1 $F_{Osc} = F_{GPS} + F_{diff}$

The frequency assigned to the house standard is the frequency of the GPS receiver plus the difference in frequency of the oscillator and the GPS based reference as measured by the counter.

2. ESTIMATED VALUES

The estimated value for the GPS receiver output is 10MHz. The estimated value for F_{diff} was calculated using the linear drift model, Eq. 2, using counter readings taken over a four month period. F_0 is the frequency deviation at an arbitrary reference time, t is the time since that reference time, and F_{drift} is the rate of drift. The results are shown in Figure 2.

Eq. 2 $F_{diff} = F_0 + t \cdot F_{drift}$

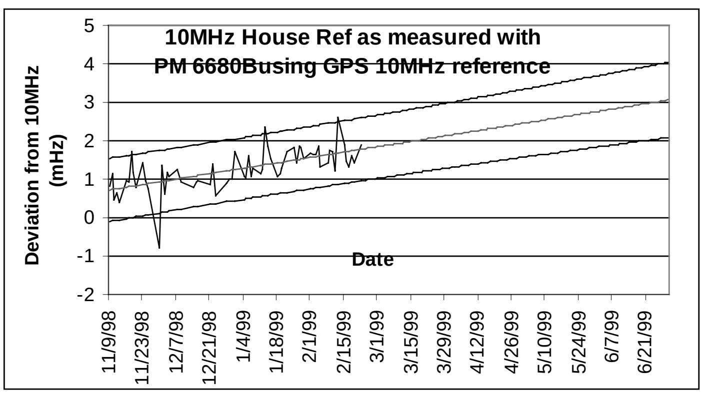

Figure 2: Measured performance of House Reference 10 MHz

Frequency readings are taken daily with the counter using a 100 sec gate time. The drift observed is +0.01 mHz per day. Having had decades to age, the drift rate is now about 50 times better than the manufacturer's maximum drift specification. The 95% confidence bands about the regression line for the data are 0.8 mHz. Since our claim is 10 mHz, we could expect the oscillator to need adjustment every couple years.

3. APPLY SIGNIFICANT CORRECTIONS

F_{diff}, the accumulated frequency error due to drift, represents a significant correction that the GUMs indicate should be applied to the measurement result. However, most of the guides also recognize that it is not always possible, practical, or desirable to make such corrections. In our case, rather than providing corrections to the Josephson Array system and calibration stations, we claim $F_{Osc} = F_{GPS} =$ 10MHz and account for F_{diff}, not in the *value* assigned to F_{Osc}, but in its *uncertainty*. As long as the uncertainty can be maintained to better than a part in 10^9, there is little advantage making the corrections. This decision will complicate the uncertainty calculation, however.

4. IDENTIFY ERROR SOURCES

The accreditation assessors are also persistent that all sources of error be identified, even those that are subsequently declared insignificant. Though it seems unprofitable, listing the uncertainties forces a conscious and documented decision as to which uncertainties to ignore and often uncovers some uncertainties which should not be ignored.

It will soon become evident that we were more successful in identifying sources of error than separating, quantifying, and attributing specific measurement results to individual sources of error.

It is not necessary at this stage to separate Type A and Type B uncertainties. It was done here, however, to save having to list them again in the paper. The error sources are grouped by the three major system components:

- **GPS System:** Includes the GPS receiver, its internal oscillator, the roof mounted antenna, satellites, and ground stations.

  Type A

  $U_{GPS\ Noise}$
  GPS oscillator output noise that may degrade the counter trigger

  $U_{GPS\ Drift}$
  Frequency drift and jitter of the GPS

  Type B

  $U_{GPS\ 24Hr\ Abs}$
  24 hour average absolute uncertainty of the GPS locked to at least 4 satellites

  $U_{GPS\ Drift}$
  Drift of the GPS system when unlocked

- **Sulzer Ovenized Oscillator**: Includes the oscillator, oven control, power supply, frequency doubler, and distribution network.

  Type A

  $U_{Osc\ Noise}$
  Noise on the oscillator signal that may degrade the counter trigger

  $U_{Osc\ jitter}$
  Short term frequency variations of the oscillator (1 to 100 seconds)

  $U_{Osc\ ST\ Drift}$
  Short term frequency drift of the oscillator (approx. 24 hours)

  $U_{Osc\ LT\ Drift}$
  Long term frequency drift of the oscillator

- **Frequency Counter**: The PM 6680B Counter is operated in external reference mode. The uncertainty analysis considers the gate time selected. The counter is operated using the math function, (K*X+L)/M to display the measured result, X, with higher resolution. The constants are set to: K=M=1 and L=-1e7 so the displayed value is the deviation from the nominal 10 MHz.

  Type A

  $U_{Cntr\ Quant}$
  Counter quantization error

  Type B

  $U_{Cntr\ Res}$
  Display Resolution

5. WRITE UNCERTAINTY EQUATION

The GUMs, at this point, recommend writing one large equation showing each uncertainty multiplied by a sensitivity factor and by correlation coefficients for the uncertainties that are not independent. For readability, a simplified form is shown below in Eq. 3

Eq. 3 $$\mathrm{U}_{Expanded} = \mathrm{U}(\mathrm{F}_{diff}) + k\sqrt{\mathrm{U}^2_{Type\_A} + \mathrm{U}^2_{Type\_B}}$$

where $U^2_{Type\_A}$ and $U^2_{Type\_B}$ are the sum of the squares of the Type A and Type B uncertainties identified in Section 4. In this analysis, all the sensitivity factors are unity and the uncertainties are treated independently so sensitivity factors and correlation coefficients are not shown. *k* in Eq. 3 is the coverage factor associated with a 95% confidence.

$U(F_{diff})$ is an uncertainty associated with the accumulated drift from Eq. 2, F_{diff}, which we are now forced to deal with in some manner.

Dealing with Uncorrected Bias

There are many benefits expressing uncertainties as Type A and Type B instead of random and systematic, but the ability to easily handle uncorrected bias is not among them. Ultimately, we are forced to express the bias as a standard deviation. The GUMs offer several suggestions for handling our "non-statistical" bias. The first three are based on the known or assigned worst-case limits for the uncorrected bias error and are explained in more detail in TAG4 [1] & Z540-2 [4], Sect. F.2.4.5.

Worst-case Method, Symmetric Limits

This method can be applied when the worst-case limits of the bias are equally spaced about the mean of the uncorrected measurement values. If so, calculate the expanded uncertainty without taking the bias into account, then add the maximum amount of the bias.

This method was selected for our analysis because it is the simplest, there is no need to reduce our uncertainty statement to the minimum defendable value, and because the bias will be a "constant" for many months. It must be recognized that this method is also the least efficient, that is, it produces the largest uncertainties associated with the uncorrected bias.

For our GPS system, the task is to determine the maximum bias (F_{diff}) we will allow before making an adjustment of the Sulzer oscillator frequency.

Worst-case Method, Asymmetric Limits

If the bias limits are not symmetric about the uncorrected output, calculate the expanded uncertainty without taking the bias into account and either:

- Add the larger magnitude to the expanded uncertainty.

OR

- Create unsymmetrical limits for the uncertainty by adding the expanded uncertainty to the positive limit and subtracting the expanded uncertainty from the negative limit for the systematic error.

Expected Value Method

In this method, a correction for the bias is made to the measurement results based, not on the value of the bias, but on its worst-case limits. Thus, the same correction is always applied. Since some of the bias is corrected, the uncertainty associated with the uncorrected portion of the bias is less than the two worst-case methods just described. It is calculated assuming a rectangular distribution between the upper and lower limits of the bias.

Eq. 4 $$\mathrm{U}(\mathrm{F}_{diff}) = \frac{UpperLimit - LowerLimit}{\sqrt{12}}$$

Corrected Uncertainty Methods

The expanded uncertainty stated using the three worst-case methods described above, will always be overstated; that is, the coverage factor provided with expanded uncertainty will correspond to the desired 95% confidence only when the uncorrected bias approaches its limits. Elsewhere, the coverage factor will describe a higher confidence.

Philips et. al. [6] investigated more efficient ways of dealing with uncorrected bias. They described three methods of using the estimated bias, not to correct the measurement result, but to correct the uncertainty. However, worst case limits could be applied to these calculations as well.

Eq. 3 was stated for the "Worst-Case with Symmetric Limits Method" we selected for this analysis. The three equations below show how Eq. 3 would be re-stated for these "Corrected Uncertainty Methods" of dealing with the uncorrected bias, F_{diff}.

RSSu$_c$ Method:

Eq. 5

$$U_{Expanded} = k\sqrt{\mathrm{U}_{Type\_A}^2 + \mathrm{U}_{Type\_B}^2 + \mathrm{F}_{diff}{}^2}$$

*RSS*U *Method:*

Eq. 6

$$U_{Expanded} = \sqrt{\mathrm{k}^2(\mathrm{U}_{Type\_A}^2 + \mathrm{U}_{Type\_B}^2) + \mathrm{F}_{diff}{}^2}$$

*SUM*U *Method:*

Eq. 7 $$+U_{Expanded} = \mathrm{F}_{diff} + k\sqrt{\mathrm{U}_{Type\_A}^2 + \mathrm{U}_{Type\_B}^2}$$
(but not < 0)

Eq. 8 $$-U_{Expanded} = -\mathrm{F}_{diff} + k\sqrt{\mathrm{U}_{Type\_A}^2 + \mathrm{U}_{Type\_B}^2}$$
(but not > 0)

Phillips et. al. [6] show that the RSSu$_c$ method tends to be a little less conservative than the worst-case method we selected but still provides higher confidence than the target with the associated penalty in uncertainty. In contrast, they show the RSSU method tends to understate the uncertainty, or provide lower confidence than the target.

They recommend the SUM*U* method of Eq. 7 and Eq. 8 and show it does an excellent job of maintaining the desired 95% confidence level for low to moderate biases and has a smaller, yet conservative impact on the confidence level for very large bias than the RSSu$_c$ and RSSU methods. When worst case limits are applied to the SUM*U* Method, the same uncertainty is obtained as the Worst-Case method used in this analysis.

6. SEPARATE TYPE A & B UNCERTS.

To save space, the uncertainties were identified as Type A and Type B when they were listed in Section 4. Determining which of the uncertainties were insignificant, which would be measured (Type A) and which would be estimated or obtained from Manufacturer's specifications (Type B) turned out to be a difficult decision. On the one hand, to substantiate our modest claims, one could hardly justify a research project into the subtleties of time and frequency. On the other hand, if we lumped most of the uncertainties into one set of direct measurements and Type B uncertainties from the manufacturer's specifications, what was the point of identifying the sources of uncertainty in minute detail?

The desire to understand the system a little better and to maintain the cleanest support for our accredited claims ultimately led us to embark on the more complex approach, to assign as many of the uncertainties as possible by direct measurement.

Manufacturer's specifications and typical performance data (normally classified as Type B) were often not used directly, but to help interpret the measurement results and assign values to individual uncertainty components. Additional measurements were made substituting a higher performance PM6681 counter for the PM6680B counter to provide further insight.

7. DETERMINE TYPE A UNCERTAINTIES

The Simple Approach

Had we opted for the less complex alternative, we would have characterized the system using three uncertainties: the Type A 100 second measurements of Figure 2, the Type B GPS receiver frequency specified absolute uncertainty, and the Type B Sulzer specified 1 second stability. The one second stability specification would be used to claim adequate performance with shorter gate times.

The same drift model of Eq. 2 and expanded uncertainty Eq. 3 would have been used. The uncertainty contribution of the uncorrected bias dominates either the simple or complex analysis. The standard deviation about the measurements about regression line is 0.4 mHz. The degrees of freedom are 63. Combining the uncertainties with a coverage factor of 2 would result in a total uncertainty of 0.8 mHz + F_{diff}.

More Detailed Direct Observations

The standard deviations of consecutive counter readings taken with gate times from 1 to 20 seconds are summarized in Table 1 and plotted in Figure 3. To eliminate the increase in standard deviation due to drift, calculations were made of pooled standard deviations of 10 readings each. The Root Allan Variance (RAV), a statistic considered more descriptive of oscillator behavior, also minimizes the drift contribution and is used in oscillator specifications. It was calculated using the approximation in Eq. 9.

Eq. 9
$$\text{RAV} \cong \frac{1}{f_{Nom}} \sqrt{\sum_{k=1}^{\text{N-1}} \frac{(f_{k+1} - f_k)^2}{2(N-1)}}$$

where:

f_{Nom} is the nominal frequency, deviation from 10MHz
f_k is the k^{th} sequential counter measurement

Gate Time (Sec)	Std Dev $x10^{-11}$	Deg. of Frdm	Pooled Std Dev $x10^{-11}$	Deg. of Frdm	Root AllanV $x10^{-11}$
1	14	46	14	44	15
2	15	45	6.8	45	8.5
5	15	38	2.6	11	3.6
10	30	43	2.4	43	2.7
20	28	43	2.2	43	1.3

Table 1: Summary of PM6680B Measurements

The standard deviation calculated agrees with the RAV calculation for 1 second gate times but is considerably larger for the larger gate times. The pooled standard deviations eliminate much of the drift effects and agree much better.

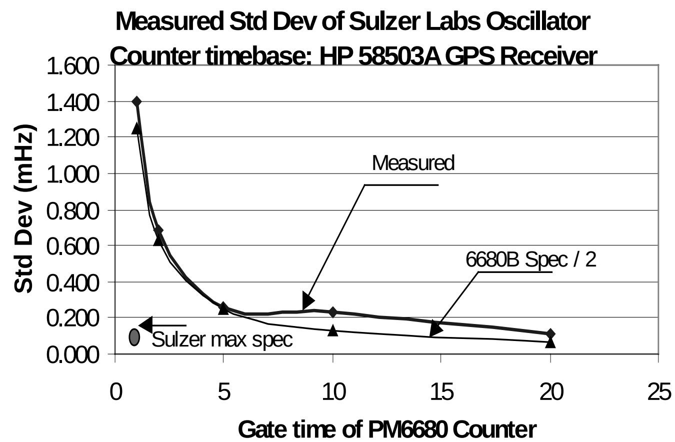

Figure 3: Measured System Performance with 1 to 20 Second PM6620B Gate Times

The measurement results immediately beg for an answer as to why they were so much larger at 1 second gate times than the Sulzer specification. For an explanation, we went to the manufacturer's specification:

Eq. 10
$$U_{cntr} = \frac{\sqrt{\text{QE}^2 + 2 \cdot \text{STE}^2}}{T} + U_{TB}$$

where:

U_{tb} = Time base uncertainty
(GPS receiver oscillator uncertainty)

T = Counter gate time

QE = Quantization Error = 250ps

$$\text{STE} = \frac{\sqrt{VN_{input}^2 + VN_{signal}^2}}{SR} \quad \text{Vrms}$$

(Start Trigger Error)

where:

VN_{input} = Internal noise = 200uV typical

VN_{signal} = RMS noise on input signal (plot assumes negligible)

SR = Slew rate of input signal, observed to be approx. $5x10^7$ V/S

The second plot in Figure 3 shows the expected standard deviation on the 6680B counter readings if it assumed the specification is derived from a coverage factor of two. As can be seen, for short gate times, the measured standard deviations correspond very closely to what would be expected from the quantization errors derived from the specifications.

To further confirm low frequency noise is due to the PM6060B counter, a PM6681 counter was substituted with the measurement results shown Figure 4 and Table 2.

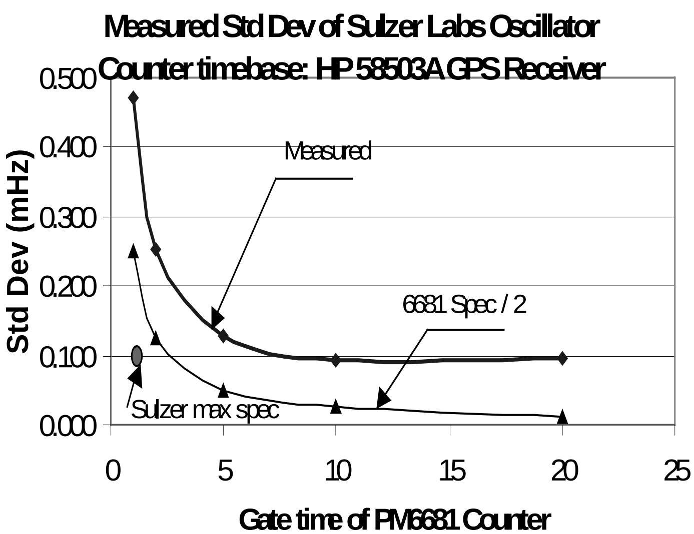

Figure 4: Measured System Performance with PM 6681 Counter

Gate Time (Sec)	Std Dev $x10^{-11}$	Deg. of Frdm	Pooled Std Dev $x10^{-11}$	Deg. of Frdm	Root AllanV $x10^{-11}$
1	6.2	96	4.7	55	3.4
2	5.5	87	2.5	42	1.9
5	6.1	84	1.3	31	1.0
10	9.8	85	0.94	35	0.72
20	43	200	0.96	106	0.64

Table 2: Summary of PM6681 Measurements

The manufacturer's specifications divided by two for the PM6681 counter are also plotted in Figure 4. They are again described by Eq. 10 but with:

QE = Quantization Error = 50ps
VN_{input} = Internal noise = 100uV typical
VN_{signal} = RMS noise on input signal (plot assumes negligible)

The measured standard deviations using the PM6681 are now larger than the PM6681 estimated uncertainty. This indicates that, in Figure 3, the increase in standard deviation as the frequency decreases is due primarily to the PM6680B quantization errors. The sources of the increased noise over the PM6681 specifications could be VN_{signal}, the Sulzer oscillator, or the oscillator in the GPS receiver that is used for the counter timebase.

The HP 58503A oscillator's noise is specified in terms of Root Allan Variance; maximum of $5x10^{-12}$, typical of about $1.5x10^{-12}$. An order of magnitude larger, $3.4x10^{-11}$ was calculated from the 1 second observations so we will not attribute this uncertainty to the GPS. However, 3mV of noise at the counter input could cause the deviations observed according to Eq. 10. Filtering the PM6681 counter input would be a reasonable experiment to see if input noise is the predominant uncertainty contributor at low frequencies. From our observations and with some help from Eq. 10 we can describe the low frequency uncertainty as being dominated by $U_{Osc\ Noise}$ = 0.47mHz/T where T is the counter gate time.

The noise ceases to decrease with longer gate times. This would indicate noise from either the GPS or Sulzer oscillators since all the counter uncertainty specifications except the reference decrease with gate time. For 20 second gate times, the HP GPS oscillator is specified for a Root Allan Variance (RAV) of $1x10^{-11}$ maximum and $5x10^{-12}$ typically. The measured Root Allan Variance was calculated to be $6.4x10^{-12}$. We will attribute this variance seen at longer gate times to the GPS oscillator.

The GPS oscillator specified RAV for 100 second samples is $5x10^{-11}$ maximum, $1.2x10^{-11}$ typically. The variation about the regression we see in the 4 month data with 100 second gate times is $4x10^{-11}$. The Sulzer 24 hour aging rate is specified as $5x10^{-11}$ maximum but we have measured it about 50 times better than that averaged over many days. It is very difficult to assign this uncertainty to either oscillator.

The dilemma is magnified by the results of an additional experiment in which the Sulzer and GPS oscillators were each compared in turn to a rubidium standard using a Frequency and Phase Comparator. RAV calculations for 100 seconds averaging time were $2.2x10^{-12}$ for the Sulzer and $9x10^{-12}$ for the GPS oscillator. Reluctantly we will assign an uncertainty of 0.4 mHz ($4x10^{-11}$) for the Sulzer short term drift based on the observations.

Though time consuming, the additional observations have given us a much better understanding of the system. They have pointed out areas to be investigated for improvement of our uncertainty claims. And finally, they disclosed that we have not yet identified the uncertainty component which would explain the larger than expected variation of the daily readings about their regression.

Values for the Type A Uncertainties

GPS System

$U_{GPS\ Noise}$ Negligible

$U_{GPS\ Drift}$ 0.09mHz ($9x10^{-12}$)

Sulzer Ovenized Oscillator

$U_{Osc\ Noise}$ 0.47mHz/T

(Negligible for T=100 sec)

$U_{Osc\ jitter}$ 0.1 mHz ($1x10^{-11}$)

$U_{Osc\ ST\ Drift}$ 0.4 mHz(($4x10^{-11}$)

$U_{Osc\ LT\ Drift}$ 0.093 mHz ($9.3x10^{-12}$)

(uncertainty of the regression)

PM6680B Counter

$U_{Cntr\ Quant}$ 250pS/T T=100

(Negligible for T=100 sec)

The significant Type A uncertainties are the long and short term drift characteristics of the Sulzer oscillator. Counters in systems using the House 10 MHz Frequency Reference and short gate times will need to be evaluated for the effect of the Sulzer oscillator noise on their performance. The significant Type A uncertainty terms are combined in Eq. 12.

Eq. 12

$$U_{Type\_A} = \sqrt{U^2_{OSC\ Jitter} + U^2_{OSC\ ST\ Drift}} = 0.42 \text{ mHz}$$

The effective degrees of freedom are about 63 as $U_{Osc\ ST\ Drift}$ dominates the uncertainty.

8. DETERMINE TYPE B UNCERTAINTIES

Type B uncertainties for the three major system components were determined from the manufacturer's specifications. Because the drift of the GPS oscillator when the system is not locked to the satellites is so large compared to the drift of the Sulzer oscillator, it is not considered. If GPS lock is lost Sulzer oscillator will be used until it is projected to be out of confidence.

GPS System

$U_{GPS\ 24Hr\ Abs}$ $1x10^{-12}$ (0.01mHz)

for a one day average

PM6680B Counter

$U_{Cntr\ Res}$ 0.01 mHz

using the (K*X+L)/M math function

(assumed rectangular distribution so

sigma = $\frac{0.01}{\sqrt{12}}$ mHz)

Eq. 13

$$U_{Type\_B} = \sqrt{U^2_{GPS\ 24Hr\ Abs} + U^2_{Cntr\ Res}}$$

$$= \sqrt{(10^{-12})^2 + (\frac{10^{-12}}{\sqrt{12}})^2} = 1.04 \cdot 10^{-12}$$

= 0.0104 mHz

The degrees of freedom for the Type B uncertainties are assumed to be large, about 200.

9. ASSIGN SENSITIVITY COEFFICIENTS

For this analysis, the sensitivity coefficients are all unity. Normally they would be written with the uncertainty expression in Section 5 and would be calculated or assigned in this section. Sensitivity coefficients are commonly encountered to make unit conversions since all the uncertainties must have the same units to combine them in the uncertainty equation.

For example, in this uncertainty analysis, we have used proportional parts and mHz somewhat interchangeably. To be strictly correct, uncertainties expressed in mHz should have been associated with a sensitivity coefficient of $1x10^{-11}$ parts/mHz.

A thermometry system may have uncertainties associated with thermal EMFs and would need a sensitivity coefficient to allow them to be expressed in terms of temperature uncertainties.

10. CALCULATE OR ASSIGN CORRELATION COEFFICIENTS

In this analysis we assumed all the uncertainties are independent. Many uncertainty analyses contain correlated inputs, however. Examples would be a micrometer calibration using a ceramic gage block. Even though they have different temperature coefficients, they both would see the same changes in temperature and would have correlated uncertainties.

Dependent uncertainties may reduce the effects of some sources of error as well. If we were to calibrate a micrometer with a gage block of made of the same metal, both the micrometer and gage block would have similar coefficients of expansion and would see nearly the same temperature. The resulting correlation coefficients would cancel much of the error associated temperature variations with a reduction in the associated uncertainty.

11. CALCULATE THE COMBINED STANDARD UNCERTAINTY

The Type A and Type B uncertainties are combined by RSS as shown in Eq. 14.

Eq. 14

$$U_{Combined} = \sqrt{(0.42)^2 + (0.0104)^2} = 0.42 \text{ mHz}$$

12. CALCULATE EFFECTIVE DEGREES OF FREEDOM

The effective degrees of freedom can be calculated using the Welch-Satterthwaite formula listed in the GUMs but will be about 63 as the Type A uncertainty dominates the combined standard uncertainty of Eq. 14.

13. OBTAIN COVERAGE FACTOR

The coverage factor is calculated using the effective degrees of freedom and the Student’s t table listed in the GUMs for 95%. In this case, since the degrees of freedom are large, we will use a coverage factor of two (k=2).

14. CALCULATE EXPANDED UNCERT.

The coverage factor of k=2 is applied to Eq. 3 to obtain the expanded uncertainty:

Eq. 14 $U_{total} = U(F_{diff}) + k \bullet 0.42 = U(F_{diff}) + 0.84$ mHz

15. REPORT THE UNCERTAINTY AND THE COVERAGE FACTOR

$U(F_{diff})$ is the uncompensated accumulated frequency error of the Sulzer oscillator measured by calculating a regression through the daily readings. The Sulzer oscillator frequency will be adjusted before the regression reaches 9 mHz. 1 part in 10^9 (10mHz) with coverage factor of 2 will be reported as the uncertainty.

Additionally, in the event of the loss of the GPS system, 1 part in 10^9 (10mHz) will continue to be claimed for the system as long as the projected regression is less than 9 mHz. The available data should be re-regressed at that time to calculate the regression and the expanded uncertainty for the projected dates.

CONCLUSIONS

The analysis dealt with several uncertainties that are a challenge to handle with classical statistics:

- Uncorrected bias error
- Allan Variance
- Display resolution

Some interesting classical statistics applied were:

- Pooled standard deviations
- Effective degrees of freedom

Two areas of difficulty of application of the GUM were encountered:

- Uncorrected bias error
- Identified but not quantified uncertainties, especially had we chosen the simplified approach.

The analysis is sufficient to support our needs of 1 part in 10^9 for the Josephson Array system and the calibrator consoles. Though it did not attain uncertainties tight enough to support high end counters, the areas needing improvements to do so were exposed.

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